NAMIBIA UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	**LEVEL:** 6
COURSE CODE: PBT602S	**COURSE NAME:** PROBABILITY THEORY 2
SESSION: NOVEMBER 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr. D. NTIRAMPEBA
MODERATOR:	Dr. D. B. GEMECHU

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil. Marks will not be awarded for answers obtained without showing the necessary steps leading to them

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

ATTACHMENTS

1. None

THIS QUESTION PAPER CONSISTS OF 2 PAGES (Excluding this front page)

Question 1 [25 marks]

1.1 Briefly explain the following terminologies as they are applied to probability theory.
(a) Power set $\mathcal{P}(S)$ [2]
(b) Boolean algebra $\mathcal{B}(S)$ [3]
(c) σ algebra [3]
(d) Discrete random variable X [3]

1.2 Show that if m is a measure on $\mathcal{B}(S)$, then cm is a measure on $\mathcal{B}(S)$, where $(cm)(A) = c.m(A)$ and $c > 0$ [4]

1.3 An investment firm offers its customers municipal bonds that mature after varying numbers of years. Given that the cumulative distribution function of T, the number of years to maturity for a randomly selected bond, is

$$F(t) = \begin{cases} 0 & \text{for } t < 1, \\ \frac{1}{4} & 1 \leq t < 3, \\ \frac{1}{2} & 3 \leq t < 5, \\ \frac{3}{4} & 5 \leq t < 7, \\ 1 & t \geq 7, \end{cases}$$

find
(a) $P(1.4 \leq t < 6)$; [3]
(b) $P(t \leq 5 | t \geq 2)$. [5]
(c) Find the median value of T [2]

Question 2 [27 marks]

2.1 State and prove **Markov** inequality. [3]

2.2 Let X be a ***continuous*** random variable with mean μ and variance σ^2. Also, let k be some positive integer. Show that $P[|X - \mu| \leq k\sigma] \geq 1 - \frac{1}{k^2}$ (i. e; Chebyshev's theorem). [12]

2.3 A dealer's profit, in units of 5000 Namibian dollars, on a new automobile is a random variable X having density function

$$f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the variance of the dealer's profit. [4]
(b) Show that Chebyshev's theorem holds for $k = 2$ with the above density function. [4]

2.4 Let X and Y be random variables with joint density function $f(x, y)$ and means μ_X and μ_Y, respectively. Show that the covariance between X and Y is given by
$\sigma_{XY} = E(XY) - \mu_X\mu_Y$. [4]

Question 3 [23 marks]

3.1 Show that the moment-generating function of a random variable X, which follows a binomial distribution with parameters n and p, is given by

$$(e^t p + q)^n$$

[5]

3.2 Let X be a random variable whose moment-generating function, denoted by $m_X(t)$, exists. Show that its third cumulant (k_3) is related to its first, second, and third moment by the following relationship $k_3 = \mu_3 - 3\mu_2\mu_1 + 2\mu_1^3$. [6]

3.3 (a) Show that the cumulant-generating function of a geometric random variable (X), with a probability of success denoted by p, is $K_X(t) = lnp + t - ln[1 - e^t(1-p)]$. [6]
(b) Use the cumulant-generating function provided above to find the variance of X. [6]

Question 4 [25 marks]

4.1 (a) Determine the value c so that the following function can serve as a probability distribution of the discrete random variable X:

$$f(x) = \begin{cases} c(x^2+4) & , x = 0,1,2,3 \\ 0 & , \text{otherwise} \end{cases}$$

[2]

(b) Assuming the value of c is $\frac{1}{30}$, find the characteristic function of X and use it to find the mean of X. [6]

4.2 Let X_1 and X_2 be independent random variables with probability density functions given by

$$f(x_i) = \begin{cases} e^{-x_i} & , x_i > 0,\ i = 1,2 \\ 0 & , \text{otherwise} \end{cases}$$

Find the:
(a) joint probability density function of $Y_1 = X_1 + X_2$ and $Y_2 = \frac{X_1}{X_1+X_2}$. [10]
(b) covariance between Y_1 and Y_2. [7]

END OF QUESTION PAPER